

# Application of Systematized Mathematical Models for the Optimization of Distribution Centers in Supply Chains

## Aplicación de Modelos Matemáticos Sistemizados para la Optimización de Centros de Distribución en Cadenas de Suministro

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### Abstract

This paper presents a mathematical allocation model for the opening and assignment of distribution centers, considering constraints such as capacity. The model includes objective functions to minimize total distance, minimize return costs, maximize distribution reliability, and maximize the reliability/distance ratio. The results demonstrate a significant improvement in key metrics: the model reduces the total distance traveled by 55% (from 970,760 to 434,918 meters), cuts return costs by 58% (from \$9,831,600 to \$4,151,500), and enhances delivery reliability by 10% (from 73.5% to 83.3%). Implementing the model improves logistics strategy by increasing customer delivery reliability and reducing product returns due to delays or unmet specifications.

**Keywords**— Reliability, Supply chain, Systematic mathematical model, GAMS, Distribution center, Optimization

### Resumen

Este trabajo presenta un modelo matemático de asignación para la apertura y asignación de centros de distribución, teniendo en cuenta la capacidad como restricción. El modelo incluye funciones objetivo para minimizar la distancia total, minimizar los costes de retorno, maximizar la fiabilidad de la distribución y maximizar la relación fiabilidad/distancia. Los resultados demuestran una mejora significativa de las métricas clave: el modelo reduce la distancia total recorrida en un 55% (de 970.760 a 434.918 metros), recorta los costes de devolución en un 58% (de 9.831.600 a 4.151.500 dólares) y mejora la fiabilidad de la entrega en un 10% (del 73,5% al 83,3%). La aplicación del modelo mejora la estrategia logística al aumentar la fiabilidad de las entregas a los clientes y reducir las devoluciones de productos por retrasos o incumplimiento de especificaciones.

**Palabras clave**— Fiabilidad, Cadena de suministro, Modelo matemático sistemático, GAMS, Centro de distribución, Optimización



## I. INTRODUCTION

The decision to create or relocate one or more facilities can be driven by various reasons, ranging from business expansion to the search for more lucrative locations. Conducting a thorough study of the different areas where distribution centers can be located and determining the possible transportation networks that can be managed can decide whether the business grows or declines, both in financial terms and market share. [1], [2]. There are various methods to make this decision today, but most apply to specific supply chains [3].

This paper presents a mathematical model that selects the locations where Distribution Centers (DCs) should be created and assigns the customers that each DC should serve, considering the level of delivery reliability to customers. All of this aims to reduce the rejection of finished products that are currently occurring, either because the product arrives late or does not meet the customer's required specifications [4].

The location of logistics facilities is one of the most critical decisions in designing an efficient supply chain. Location optimization not only affects transportation and storage costs but also directly influences a company's ability to respond to market needs in a timely and reliable manner [5]. The choice of strategic locations for distribution centers can significantly improve supply reliability, reduce operational costs, and increase customer satisfaction [6].

The primary objective of this work is to develop a systematic mathematical model to determine the optimal locations for distribution centers and efficiently allocate customers to these centers. This model takes into account multiple constraints and objectives, including center capacity, minimizing the total distance traveled, reducing return costs, and maximizing distribution reliability [7], [8].

To address this problem, a multi-objective approach seeks to balance the different stated goals. Four main objective functions are considered: minimizing the total distance, minimizing return costs, maximizing distribution reliability, and maximizing the reliability-to-distance ratio. Each of these functions provides a different perspective on how to optimize the distribution network, and their combination allows for balanced and robust solutions [9].

The proposed model is validated using real data from a supply chain in the agribusiness sector in the Northern region of Colombia. This supply chain is characterized by the distribution of products from a processing plant to several owned and third-party stores. Third-party stores represent more than 60% of the company's total income, making the systematic optimization of the distribution network crucial to improving reliability and reducing returns [10].

Supply chain reliability is assessed using a system that inputs and analyzes each store's sales and returns history. Returns are generated for various reasons, including delivery delays, unsought products, and incorrect specifications. Implementing distribution centers enables timely inspection and delivery, eliminating many of the causes of returns and thus improving delivery reliability [11].

The developed mathematical model includes a set of constraints and variables representing the system's different decisions and parameters. Among the main constraints are the maximum capacity of the distribution centers, the satisfaction of demand from each store, and the balance of distribution between the factory and the stores. The model variables include decisions on the opening of distribution centers, the quantities of products shipped, and distribution reliability [12].

To solve the model, GAMS (General Algebraic Modeling System) software is used, which allows the formulation and solution of complex mathematical programming problems. The model is solved using the SCIP solver, which specializes in non-linear programming problems with binary variables. The results show significant improvements in supply chain reliability, reductions in return costs, and reductions in total distance traveled [13].

In conclusion, this work presents a robust and efficient methodology for optimizing the distribution network in a supply chain. Implementing the proposed model allows companies to improve their logistics strategy, increase customer delivery reliability, and reduce operating costs. The scenarios presented offer solutions that can be adapted according to each company's specific objectives, providing a valuable tool for decision-making in supply chain design [14], [15].

## II. RELATED WORKS

The optimization of DC locations and the efficient allocation of customers are key topics in supply chain management, especially when aiming to maximize efficiency and reliability. Over the years, various researchers have addressed this challenge from different perspectives, offering innovative solutions that have significantly advanced this field.

Firstly, [16] proposed an approach to improve supply chain efficiency by optimizing DC locations. This work highlights how strategic location can reduce storage and transportation costs, essential to maintaining competitiveness in today's market. Similarly, [2] developed a mixed-integer mathematical model that optimizes the locations of distribution centers and demonstrates how these decisions can considerably reduce operational

costs. Through a case study in a construction products company, [2] showed a 20% reduction in transportation costs, highlighting the practical impact of good location planning.

On the other hand, [9] focused on the cold chain, proposing a hybrid algorithm for the location of distribution centers. This approach is particularly relevant for perishable products, where precision and speed in delivery are critical. The hybrid algorithm developed improves the accuracy and speed of convergence in solving the problem, proving its effectiveness in contexts where time is a determining factor.

[1] took a different perspective by combining genetic algorithms with queuing theory and stochastic optimization to address location inventory in supply chains. This approach allows for managing stochastic customer demand while optimizing DC locations and inventory policies. These advanced methods effectively address the complexity of the problem, providing robust solutions for complex supply networks.

Next, [13] integrated geographical analysis into optimizing distribution center locations using a hybrid model that combines particle swarm optimization and genetic algorithms. This work emphasizes the importance of considering geographic variables to improve the accuracy of DC location, which is crucial for large supply chains.

Additionally, [6] introduced a two-level stochastic optimization model designed to operate in competitive environments. This model optimizes the distribution center's location and considers the competition between different supply chain actors, adding a layer of realism and applicability to their proposal.

In a more recent approach, [17] developed a specific method to determine the optimal coordinates for the location of distribution centers along a highway segment. This study is particularly useful for companies operating in linear transportation networks, offering a practical solution that improves reliability and reduces delivery costs.

Moreover, [8] used an affinity propagation clustering algorithm (AP) to optimize the location of centers in the cold chain. This approach is innovative as it simplifies location selection and improves the efficiency of perishable product distribution, a crucial aspect of modern logistics.

Finally, [8], [18] and [19] addressed supply chain optimization from hierarchical and uncertainty perspectives, respectively. While [18] focused on the pharmaceutical industry, proposing a hybrid algorithm to manage location and routing under uncertain conditions, [19] explored location-allocation problems in three-level supply networks under uncertainty, applying their methodology to a case study in the tea supply chain in Shanghai.

The works above demonstrate the diversity of approaches available to solve the problem of distribution center location and customer allocation. Each of these studies offers innovative and practical solutions that can be adapted to different contexts, depending on the specific needs of the supply chain. This wealth of approaches underscores the importance of careful planning and the need to use advanced mathematical models to achieve optimal results in supply chain management.

### III. METHODOLOGY

The methodology used in the study is described below:

#### A. Study Approach:

This study employs a quantitative approach based on mathematical modeling to optimize the location of DCs within a supply chain in the agribusiness sector of the Northern region of Colombia. The main objective is to evaluate the results obtained by minimizing the total distance traveled, reducing costs associated with returns, maximizing distribution reliability, and maximizing the reliability-to-distance ratio.

#### B. Data Used:

The model is based on real data obtained from a supply chain that distributes products from a processing plant to both company-owned and third-party stores in the department of Atlántico, Colombia. A total of 9 company-owned stores and 110 neighborhood stores were considered. The key data include:

**Capacity of the DCs and the factory:** Measured in daily tons.

**Distance between distribution points:** Calculated using the formula where K is the conversion factor to kilometers [20]:

$$d_{p1-p2} = K \sqrt{(lon_{p2} - lon_{p1})^2 + (lat_{p2} - lat_{p1})^2} \quad (1)$$

**Sales and Returns History:** Used to calculate the reliability of the supply chain and determine the impact of returns on reliability.

### C. Mathematical Modeling:

A multi-objective mathematical programming model was developed to select the optimal locations for DCs and efficiently allocate customers to these centers. The model considers four objective functions:

**Minimizing total distance traveled:** This function aims to reduce distance between distribution points to minimize transportation costs.

**Minimizing return costs:** It reduces the economic losses associated with product returns.

**Minimizing distribution reliability:** It increases the average reliability of deliveries, calculated based on the proportion of correctly delivered products versus returns.

**Minimizing the reliability-to-distance ratio:** This indicator combines distribution reliability with distance efficiency, seeking an optimal balance.

### D. Constraints and Variables:

The model includes several key constraints to ensure the feasibility and efficiency of the solution:

**Maximum DC capacity:** This is limited by the available capacity at the company-owned stores, which could act as distribution centers.

**Demand satisfaction:** Ensures that the demand for each neighborhood store is fully met, either directly by the factory or through a DC.

**Distribution balance:** Ensures that the amount of product sent from the factory to the DCs is proportional to the amount distributed from these DCs to the neighborhood stores.

**Opening of distribution centers:** The decision to open DCs is modeled using binary variables that indicate whether a company-owned store should act as a DC.

### E. Model Solution:

The model was solved using the GAMS (*General Algebraic Modeling System*) software with the SCIP (Solving Constraint Integer Programs) solver, specializing in nonlinear programming problems with binary variables. Different optimization scenarios were proposed and solved to evaluate the model's performance under different priorities, such as minimizing distance, return costs, maximizing reliability, and the reliability-to-distance ratio.

### F. Model Validation:

The validation was conducted by comparing the optimized model's results with a baseline scenario without distribution centers. The results demonstrated significant improvements in the supply chain's efficiency and reliability.

## IV. RESULTS AND ANALYSIS

**Case Study:** The case study was conducted on a company's supply chain in the agribusiness sector in the Northern region of Colombia, specifically for the segment where the product is distributed from the processing plant to the customer (stores).

The factory supplies two types of stores: owned by the company and third-party stores, representing more than 60% of the company's total income. The model only considers the stores located in the department of Atlántico, Colombia, totaling 9 company-owned stores and 110 neighborhood stores.

The factory has an established reliability rate of 87%, meaning that out of 100% of total demand, only 87% is dispatched by the logistics area. This occurs due to various internal operational issues. To determine customer reliability, the sales and returns history is used, calculated with the following equation:

$$Reliability = 1 - \frac{Returns}{Sales + Returns} \quad (2)$$

Returns are generated for various reasons, but some, such as delays, unsought products, and inadequate weight, among others, can be prevented with the help of a distribution center, as it allows for timely inspection and delivery to eliminate returns caused by these issues.

Formula 1 is used to determine the benefit of these distribution centers. This eliminates returns and converts them into sales. The reliability of customers is then recalculated, providing both current and future reliability for each customer.

For purely economic reasons, it was decided that no investment should be made in new facilities to create distribution centers. However, it was detected that the company-owned stores have idle capacity that can be used to supply nearby neighborhood stores.

On the other hand, road damage, closures, traffic jams, natural phenomena, and other factors are some of the issues that hinder the delivery of finished products from the factory to the customer. For this reason, having a distribution center that stocks products to meet some customers' demands will help prevent problems on a route from affecting the company's sales.

The company distributes an average of 40 tons/day to neighborhood stores. All stores are supplied by 5- and 10-ton trucks departing from the factory.

The model must establish the distance between the factory and company-owned stores, the factory and neighborhood stores, and the company-owned stores and neighborhood stores. For this, the "Unspecified Source" formula is used.

$$d_{p1-p2} = K \sqrt{(lon_{p2} - lon_{p1})^2 + (lat_{p2} - lat_{p1})^2} \quad (3)$$

$K$  is the conversion factor from degrees to miles (69) or kilometers (111.319),  $lon$  is the longitude, and  $lat$  is the latitude of the points studied. This equation is suitable since it does not consider the earth's curvature.

The route costs are directly proportional to the distance traveled, so we minimize the costs of transporting the goods by minimizing distance.

Finally, it is established that each customer's demand is the sum of sales and returns. With all these parameters, the mathematical model is formulated using different approaches and scenarios.

**Systemic Mathematical Modeling:** To determine the optimal locations for the consolidation centers, a mathematical programming model is developed with four different objectives: minimizing distance, minimizing return costs, maximizing reliability, and maximizing the reliability-to-distance ratio. The following model is described in general terms to facilitate its use in future studies:

#### **Sets**

I = Company-Owned Stores (Distributor)

J = Neighborhood Stores (Customers)

#### **Parameters**

Cf = Factory Reliability

Cftp = Factory to Company-Owned Stores Reliability

Cftb = Factory to Neighborhood Stores Reliability

Ctptb = Company-Owned Stores to Neighborhood Stores Reliability

CPtp = Company-Owned Store Capacity

DEtb = Neighborhood Store Demand

Dftp = Distance between Factory and Company-Owned Stores

Dftb = Distance between Factory and Neighborhood Stores

Dtptb = Distance between Company-Owned Stores and Neighborhood Stores

CDftb = Return Costs from Factory to Neighborhood Stores

CDtptb = Return Costs from Company-Owned Stores to Neighborhood Stores

N = Total Number of Neighborhood Stores

#### **Variables**

Xtptb = Decision if Company-Owned Store serves Neighborhood Store

Xftb = Decision if Factory serves Neighborhood Store

Xftp = Decision if Factory serves Company-Owned Store

Qftp = Number of Kilos from Factory to Company-Owned Store

Qftb = Number of Kilos from Factory to Neighborhood Store

Qtptb = Number of Kilos from Company-Owned Store to Neighborhood Store

Con = Neighborhood Store Sales Reliability

### OBJECTIVE FUNCTION

Four different objective functions are established, as each may provide a solution that can be considered, comparing total distance, reliability, and the savings each offers.

**Minimization of Distance (Meters):** This function is divided into three summations. The first accounts for the distance from company-owned stores to neighborhood stores, the second the distance from the factory to neighborhood stores, and the third the distance from the factory to company-owned stores:

$$Z_{min} = distancia = \sum_i \sum_j Xtptb_{ij} * Dtptb_{ij} + \sum_j Xftb_j * Dftb_j + \sum_i Xftp_i * Dftp_i \quad (4)$$

**Minimization of Return Costs (\$),** This function seeks to minimize the money lost due to returns. To achieve this, we sum the return costs incurred if the factory serves the store plus the return costs incurred if a consolidation center serves the store:

$$Z_{min} = costos = \sum_j Xftb_j * CDftb_j - \sum_i \sum_j Xtptb_{ji} * CDtptb_{ji} \quad (5)$$

**Maximization of Reliability (%),** This function aims to maximize the average customer reliability. Each customer's reliability is defined by whether they are served by a consolidation center or by the factory. Since only one can serve the customer, one value will be 0, and the other will define the reliability. Then, all the values for each customer are multiplied, and the root of the number of data points is taken:

$$Z_{max} = confiabilidad = \sum_j \frac{(\sum_i (Xtptb_{ji} * Ctptb_{ji} * Cftp_i) + Xftb_j * Cftb_j) * Cf}{N} \quad (6)$$

**Maximization of Reliability/Distance (Indicator %/meters),** This function seeks a balanced solution with good reliability and total distance:

$$Z_{max} = \frac{Reliability}{Distance} \quad (7)$$

### Constraints

**Demand Satisfaction** is the sum of the quantities sent from all company-owned stores to the neighborhood stores (multiplied by the binary variable indicating whether the store serves the neighborhood store) plus the quantities sent from the factory to the neighborhood stores (multiplied by the binary variable indicating whether the factory serves the neighborhood store) must be equal to the demand of each neighborhood store, for all neighborhood stores.

$$\sum_i Qtptb_{ji} * Xtptb_{ji} + Qftb_j * Xftb_j = DEtb_j \quad \forall j \quad (8)$$

**Maximum Capacity of Company-Owned Stores:** the quantity sent from the factory to the company-owned store for distribution to neighborhood stores must be less than or equal to the available capacity of the company-owned store multiplied by the binary variable that indicates if the distribution center is opened.

$$Qftp_i \leq CPftp_i * Xftp_i \quad \forall i \quad (9)$$

**Distribution Balance of Company-Owned Stores:** the sum of the quantities sent from the company-owned stores to the neighborhood stores must equal the quantities the factory sent to the company-owned stores.

$$\sum_j Qtptb_{ji} = Qftp_i \quad \forall i \quad (10)$$

Opening of Distribution Centers, the binary variable indicating whether the factory sends to a company-owned store must be greater than or equal to the binary variable indicating whether the company-owned store sends to the neighborhood store. This applies to all company-owned stores and all neighborhood stores.

$$Xftp_i \geq Xtptb_{ji} \quad \forall i, \forall j \quad (11)$$

Only One Company-Owned Store or Factory Serves a Neighborhood Store. For all neighborhood stores, the sum of the binary variables indicating whether the company-owned stores serve a neighborhood store plus the binary variable indicating whether the factory serves the neighborhood store must equal 1.

$$\sum_i Xtptb_{ji} + Xftb_j = 1 \quad \forall j \quad (12)$$

Factory Capacity: the sum of the quantities sent from the factory to the company-owned stores plus the sum sent from the factory to the neighborhood stores must be less than or equal to the production capacity.

$$\sum_i Qftp_i + \sum_j Qftb_j \leq CPf \quad (13)$$

Reliability of Distribution per Neighborhood Store, defined by the factory's reliability, multiplied by the sum (for all company-owned stores) of the product between the binary variable indicating whether the company-owned store sends the product to the neighborhood store and the reliability of the neighborhood store if served by the company-owned store, and the reliability of the company-owned store if served by the factory. Plus, the binary variable indicates whether the product is sent to the neighborhood store from the factory, multiplied by the reliability of the neighborhood store if served by the factory. This applies to all neighborhood stores.

$$con_j = \left( \sum_i (Xtptb_{ji} * Ctptb_{ji} * Cftp_i) + Xftb_j * Cftb_j \right) * Cf \quad \forall j \quad (14)$$

Total Distance Traveled, added as a constraint to calculate the distance traveled for each possible solution.

$$a = distancia \quad (15)$$

Average Reliability is a constraint to calculating the reliability of each possible solution.

$$b = \sum_j \frac{con_j}{N} \quad (16)$$

Savings are added as a constraint to calculate the savings for each possible solution.

$$c = costos \quad (17)$$

The reliability-to-distance ratio was added as a constraint to calculate the reliability-to-distance ratio for each possible solution.

$$d = \frac{b}{a} \quad (18)$$

**Binary Variable Constraints:** the decision variables are limited to values of 0 or 1.

$$Xtptb_{ji}, Xftb_j, Xftp_i = \{0,1\}; \forall i \forall j \quad (19)$$

**Non-Negativity Constraint,** all variables in the problem are limited to positive numbers only.

$$Xtptb_{ji}, Xftb_j, Xftp_i, Qtptb_{ji}, Qftb_j, Qftp_i, a, b, c, d \geq 0; \forall i \forall j \quad (20)$$

**Initial Model Results:** The current model, in which all stores are served by the factory and there are no distribution centers, is created to establish a starting point in terms of costs, distance, and reliability. To determine these comparative values, the capacity of the distribution centers was limited to 0, and the following results were obtained.

This model and the scenarios in Section 6 were solved using the GAMS program, utilizing the SCIP solver, and solving them as nonlinear programming models with binary variables.

Table 1. Initial Model Results.

CURRENT MODEL	VALUES
Total distance	970,760
Return cost	\$ 9,831,600
Reliability	73.5%
Reliability/distance ratio	7.5651

Source: Author

### **Proposed Model Results**

**Scenario 1: Minimizing Distance:** The results of this scenario show the opening of 6 out of the 9 distribution centers, distributing 4,500 Kg, as shown below:

Table 2. Kilos are sent from the factory to distribution centers, with the scenario minimizing distance.

DISTRIBUTION CENTER	KILOS
1	0
2	900
3	2550
4	330
5	0
6	270
7	210
8	0
9	240

Source: Author

The values of the comparative variables are as follows:

Table 3. Results of the scenario minimizing distance.

MINIMIZE DISTANCE	VALUES
Total distance	434,918
Return cost	\$ 4,151,500
Reliability	81.7%
Reliability/distance ratio	0.002

Source: Author

**Scenario 2: Minimizing Return Costs:** To minimize costs, 8 out of the 9 distribution centers are opened, distributing 4,980 Kg, as shown below:

Table 4. Kilos are sent from the factory to distribution centers, with the scenario minimizing return costs.

DISTRIBUTION CENTER	KILOS
1	270
2	0
3	2550
4	330
5	840
6	270
7	210
8	270
9	240

Source: Author

The values of the comparative variables are as follows:

Table 5. Results of the scenario minimizing return costs.

MINIMIZE RETURN COST	VALUES
Total distance	1,010,564
Return cost	\$ 2,609,100
Reliability	83.3%
Reliability/distance ratio	8.2312

Source: Author

**Scenario 3: Maximizing Reliability**

To maximize reliability, 8 out of the 9 distribution centers are opened, distributing 5,670 Kg, as shown below:

Table 6. Kilos sent from the factory to distribution centers, scenario maximizing reliability.

DISTRIBUTION CENTER	KILOS
1	270
2	900
3	2550
4	330
5	840
6	270
7	0
8	270
9	240

Source: Author

The values of the comparative variables are as follows:

Table 7. Results of the scenario maximizing reliability.

MAXIMIZE RELIABILITY	VALUES
Total distance	1,416,849
Return cost	\$ 2,609,100
Reliability	83.3%
Reliability/distance ratio	5.8726

Source: Author

**Scenario 4: Reliability-to-Distance Ratio**

To maximize the reliability-to-distance ratio, 6 out of the 9 distribution centers are opened, distributing 1,244.54 Kg, as shown below:

Table 8. Kilograms shipped from the factory to the distribution centers, scenario maximizing reliability over distance ratio.

DISTRIBUTION CENTER	KILOS
1	0
2	159,7
3	169,15
4	195,69
5	0
6	270
7	210
8	0
9	240

Source: Author

The values of the comparative variables are as follows:

Table 9. Results of the scenario maximizing the reliability-to-distance ratio.

MAXIMIZE RELIABILITY-TO-DISTANCE RATIO	VALUES
Total distance	437,208
Return cost	\$ 3,660,400
Reliability	82.4%
Reliability/distance ratio	0.002

Source: Author

**Analysis of Results:** The proposed scenarios present solutions that may or may not be optimal, depending on the company's objectives. For this reason, these scenarios are compared for each of the objective variables.

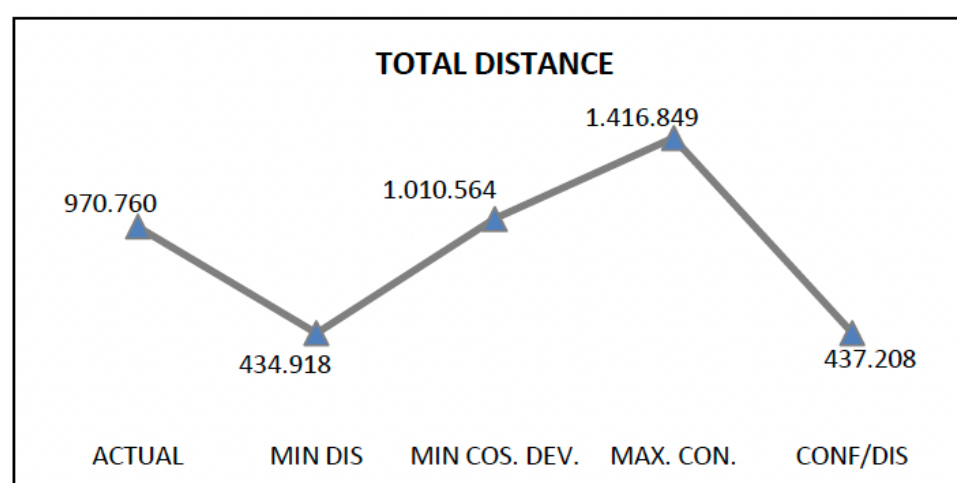


Figure 1. Results of the total distance across scenarios.

Source: Author

In Figure 1, the shortest total distance traveled is achieved when the objective is to minimize distance. Notably, a solution that is very close to the optimal distance is obtained when the objective is reliability over distance. It is important to highlight that the other two solutions result in a distance twice as long as the previous two.

The second variable to analyze is the return cost, as knowing how many lost sales are generated by returns for each scenario allows the company to make decisions regarding strategies to prevent these from occurring or, if it is too costly to cover them, to determine if it is better to accept the losses.

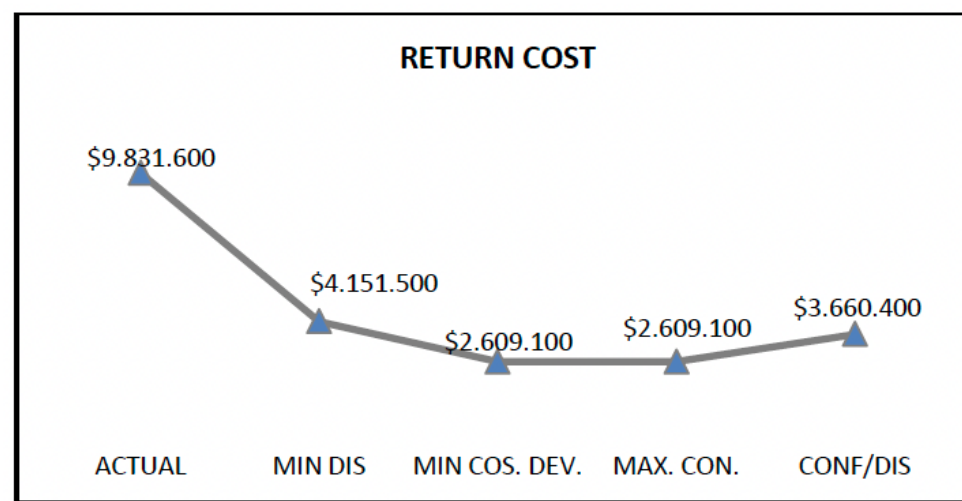


Figure 2. Results of the return cost across scenarios.

Source: Author

In Figure 2, the lowest return cost is achieved when minimizing this cost and maximizing reliability. The scenario closest to these is when the reliability-to-distance ratio is evaluated, with a difference of just over 1,000,000 Colombian pesos.

The last variable to analyze is the average reliability of the routes, which provides the reliability of the final part of the supply chain (factory-to-customer).

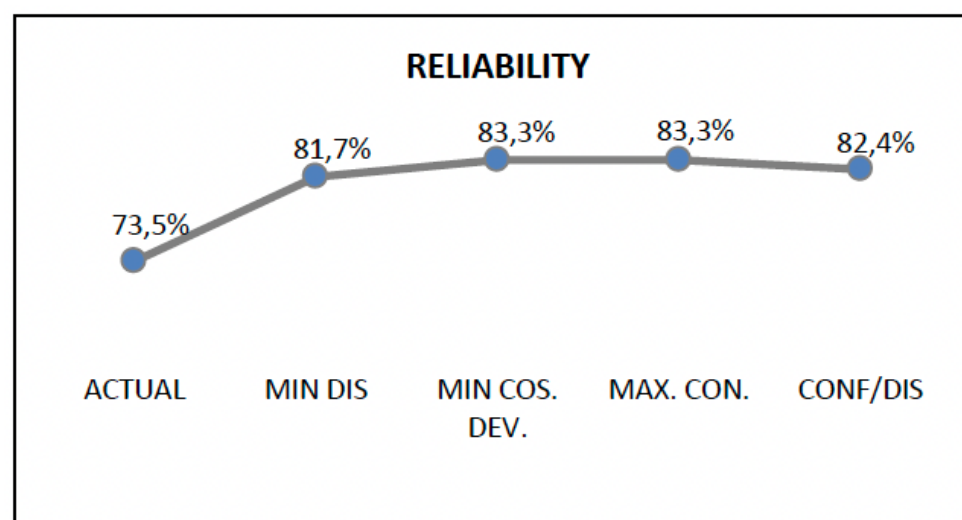


Figure 3. Results of the reliability across scenarios.

Source: Author

In Figure 3, the highest reliability is achieved both when maximizing it and when minimizing return cost. The reliability closest to these is obtained when evaluating the reliability-to-distance ratio, with a difference of 0.9% from the optimal value of 83.3%.

## V. CONCLUSIONS

As a result of the systematized mathematical model presented in this document, the assignment of [21] possible distribution centers to be enabled is determined to improve the company's logistical performance [22].

The proposed scenarios offer solutions that allow [23] for more than a 50% reduction in initial return costs and an improvement in the reliability of the supply chain by approximately 8 to 10%.

Among the scenarios, it is notable that the fourth scenario, which aims to maximize the reliability-to-distance ratio, presents solutions that are very close to the optimal for the three variables of analysis [16]. Another point worth highlighting is that in this scenario, only about 1245kg is distributed, which is not even half of what would need to be distributed in the other scenarios [24]. This indicates that priority is being given to distributing to the customers who represent the greatest benefit when covered by a distribution center [25].

The results of each scenario ultimately provide the company with different alternatives from the perspectives of cost, reliability, or the reliability/cost ratio [26], allowing it to determine how to distribute its products to neighborhood stores [27].

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## AUTHORS' CONTRIBUTION

The authors' contributions to this article are as follows:

Luis Enrique Patiño-Toledo: Research, writing, Software development, review, and editing.

Dairon Jesús Torrado-Castro: Software development, data analysis, visualization and writing, review, and editing.

All authors participated in the review of the results and gave their approval to the final version of the manuscript.

## CONFLICT OF INTERESTS

The authors hereby declare that there are no conflicts of interest pertaining to the reporting of this study.

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