Algoritmo de optimización multiobjetivo de una región contaminada

A multi-objective optimization of contamination algorithm for a contaminated region

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Resumen

Introducción: En este trabajo se considera un problema de la ecología industrial bajo el enfoque de optimización semi-infinita.
Objetivo: El objetivo es la resolución del conflicto entre las emisiones contaminantes con las normas ambientales para las zonas de una región dada.
Metodología: Se propone una versión del algoritmo SIP2 que, simultáneamente con la disminución de la contaminación, también permite un aumento de las emisiones de las fuentes, de tal forma que permite una cierta libertad en el manejo de contaminación y los elementos que la provocan en la industria, pero velando por el cumplimiento de la normatividad ambiental.
Resultados: Los resultados de dos algoritmos muestran diferentes papeles de las fuentes en la obtención de la contaminación total y como consecuencia la necesidad de los cambios en sus emisiones.
Conclusiones: El algoritmo propuesto ofrece soluciones más rentables en evaluaciones de diseño de infraestructura para áreas con intereses opuestos, como el mantenimiento o el aumento de la producción (medido indirectamente por la generación de emisiones debido al sistema productivo), al tiempo que garantiza el cumplimiento de las normas ambientales restrictivas.
Palabras claves: región contaminada; emisiones; normas de contaminación; optimización semi-infinita; toma de decisiones.

Abstract

Introduction: This work considers an industrial ecology problem under the framework of semi-infinite optimization.
Objective: The objective is to resolve the conflict between pollutant emissions and environmental standards for the areas within a given region.
Methodology: A version of the SIP2 algorithm is proposed, which simultaneously allows for a reduction in pollution while also enabling an increase in emissions from sources. This approach provides a certain degree of flexibility in managing pollution and the factors that cause it in the industry, while ensuring compliance with environmental regulations.
Results: The results of two algorithms show different roles of the sources in obtaining the total pollution and, therefore, the need for changes in their emissions.
Conclusions: The proposed algorithm offers more cost-effective solutions in infrastructure design evaluations for areas with conflicting interests, such as maintenance or increasing production (indirectly measured by emissions generated from the production system), while ensuring compliance with restrictive environmental regulations.
Key words: contaminated region; emissions; pollution norms; semi-infinite optimization; decision making.

I. INTRODUCTION

The industrial ecology of a region usually involves many sources of pollution, whose emission parameters (power, understood as the amount of pollutant emitted per unit of time), location coordinates, affected zone, etc., vary significantly. In the area, even when there are facilities or zones that comply with environmental and safety standards, conflicts may arise that require adjustments (optimization) in the characteristics of those involved. The objective is to avoid a completely restrictive approach based solely on environmental regulations, as this could go against productivity. Instead, the aim is to find a balance in which the possibility of increasing emissions' value in certain sites or facilities is evaluated (while being aware that this is related to higher production and profitability), as long as overall environmental protection standards in the area are met.

The pollution map of a studied area corresponds to the superposition of local emissions from sources, and the function that represents them may have several extremes. Generally, the environmental regulations map is not used, but rather the vector of standards for the constituent zones of the area, whose boundaries may not necessarily have the correct forms. Variations in the values of source and zone parameters at many points in the area make conflict management challenging.

Comparing both maps (pollution and standards) divides the area into two sets of points: those with excess pollution and those with a margin of compliance regarding the standards. A reduction in both types of violations leads to a decrease in conflicts.

The optimization problems related to conflicts depend on the pollution criteria of the area. In cases of integral criteria, direct local optimization algorithms are used (e.g., Hook-Jeeves, Nelder-Mead, or others), as stated by the Authors [1]. However, such solutions may allow some non-compliance with standards in groups of points.

Ensuring compliance with pollution standards for each area point requires more complex algorithms (e.g., semi-infinite optimization (SIP) described by Vaz and Ferreira [2]; Goberna [3]), including stochastic search, local maximization procedures, and nonlinear programming (NLP). Applications of SIP to industrial ecology problems, even for point, linear, and 3D area emission sources, have been documented by [4] and [5].

In optimization applied to this type of problem, it is common to look for only excessive pollution in certain zone points and restrict the emissions from corresponding sources [2], [4], [5]. However, in many cases, these decisions are challenging to apply in areas with established infrastructure and may not always be economically viable. Therefore, multidirectional optimization becomes relevant, in which, along with the necessary reduction for one set, an admissible increase in atypical values for another set of points in the region is sought. Both changes adjust the overall pollution function to area standards. The algorithm's task is to combine these changes across the entire set of points in the region, providing a more cost-effective solution for industrial potential.

The goal of this manuscript is to propose a multidirectional optimization *SIP2* algorithm that works with a model of parabolic dispersion pollution and can be applied to find more cost-effective options for the design or reconstruction of areas with industrial facilities and protected areas. The programming, numerical experiments, and graphics are performed in MATLAB.

II. PROBLEM DEFINITION AND METHOD

In a given area where different industrial facilities generating emissions are located, a conflict arises between increasing production and complying with environmental standards. Moreover, it is impossible to control the pollution of an individual point in the area using emission vectors from sources since changes in the vector affect the pollution of many other points (subareas) simultaneously. Therefore, the possibility of drastically reducing pollution in particularly prominent subareas may lead to the emergence of others that do not comply with environmental standards or have notable margins of compliance with permitted values. In such cases, it is advisable to compensate (increase) the capacity of some sources, meaning to utilize the opposite direction of optimization.

Model variables. In a two-dimensional region $\boldsymbol{d}$, there are N point sources emitting pollutants with power $\boldsymbol{P}$ and coordinates $\boldsymbol{tN}$. The area is divided into W zones with curvilinear boundaries and a pollution standard vector for the zones, $\boldsymbol{Norm}$.

Dispersion Model. A comprehensive interpretation of conflicts in industrial ecology includes pollution dispersion models used in the algorithms to optimize area pollution. In this work, a parabolic dispersion model for pollutants was adopted. While it may seem arbitrary, it is geometrically easy to formalize and highly suitable for the development of optimization algorithms. In future work, it may be replaced by a more complex model or one considered more appropriate.

In this model, the volume of the paraboloid $P = \pi H^2/2a$ (where H is the ordinate of the pollutant source and a is the dispersion coefficient) corresponds to the integral of all emissions over a defined period.

Pollution Zone. Let $h_{s,j}$ be the amount of pollution that reaches point s with coordinates [*X, Y*] from pollution source:

$$h_{s,j} = -a_j r_{s,j}^2 + H_{\boldsymbol{j}},$$

where: H_j is the ordinate of pollutant source j and **r** is a distance matrix from sources to point s:

$$r_{s,j} = \sqrt{(tN_{1,j} - X_s)^2 + (tN_{2,j} - Y_s)^2} \ ,$$

where: $[j = 1 \dots N]$ are the indices of the set of pollution sources. The $tN_{1,j}$ and $tN_{2,j}$ are the coordinates of source j.

A source is considered active for a point s in the area if, for the maximum allowed pollution value r_j^{max} the inequality $r_{s,j} < r_j^{max} = \sqrt{H_j/a_j}$ is satisfied.

SIP Algorithm. The SIP problems are optimization problems with an objective function over an infinite set of constraints (1):

$$f_u \rightarrow \min, \qquad g_{u,s} \leq 0, \qquad (1)$$
$$0 \leq u \leq 1$$

To solve it, the semi-infinite optimization problem is replaced by a sequence of finite problems, and each of them considers only the relevant (active) constraints from the initial infinite set of constraints.

To find these constraints s*, the constraint function g is maximized, seeking values of s such that $g_{u,s} > 0$. The starting point for local maximization is found using a uniform distribution. The generated NLP (Nonlinear Programming) matrix determines the (approximate) solutions of the vector u and only in one direction, making the optimization considered unidirectional. For more details, see, for example [3].

New version of the algorithm. In the *SIP2* algorithm version, the optimization parameter remains the $\boldsymbol{u}$ a selective decrease in the ordinate $\boldsymbol{H}$, depending on the activity of the sources spreading pollution. However, changes in $\boldsymbol{u}$ are allowed in both directions, which also nonlinearly alters the volumes of the original dispersion paraboloids.

The choice of the optimization direction is made through the condition:

$$g_{u,s} = \sum_{j=1}^{N}\left(-a_j r_{j,s}^2 + (1-pp)H_j\right) - Norm_s \leq 0 \quad (2)$$

$$pp = u_j \text{ si } pre_{j,tN} \geq 0$$

and $pp = -do$ in another case,

$$pre_{j,tN} = H_j - Norm_{j,tN} \qquad (3)$$

here, the excess pollution H_j above the norm $Normt_{j,tN}$ in the area is due to emissions from source j; do – is the value of power addition to source $\boldsymbol{u}$ (the negative sign of do corresponds to the selective increase for some sources in constraint (2)).

According to expression (3) the set of sources $B = \{j | pre_{j,tN} < 0\}$, $B \in A$, $A = \{j | 1, \dots, N\}$, is obtained, for which the algorithm performs the opposite optimization.

The algorithm finds the value of B at the beginning and keeps it in the following iterations. The simplicity of $pre_{j,tN}$ is sufficient to illustrate the condition to resort to the opposite direction of optimization in (2).

Next, it is necessary to determine the value of the permissible increase of the elements in the solution **u** for the subset *B*. For better clarity, the value of do is assumed to be fixed, but its choice can be a function of many parameters such as excesses, number of iterations, coefficients associated with economic considerations, or others. The proposed algorithm not only indicates reductions in the sources but also allows for the conservation of their allowed powers, managing the structure of the solution $\boldsymbol{u}$ in this way.

The multidirectional optimization simultaneously "bends" the overall pollution function of the area not only from above but also from below. The criterion for terminating the optimization process is the absence or minimization of excesses according to the area's **Norm** standards.

In global optimization, there are techniques that reform an iterative solution outside the main algorithm. The use of hybrid methods can be quite complex, including the resolution of their own sub-optimization problems [6].

The objective function (4) of the NLP (Nonlinear Programming) procedure minimizes the summation of costs $\boldsymbol{c}$ of source powers according to the solution $\boldsymbol{u}$

$$C_u = \pi \sum_{j=1}^{N} c_j \left(u_j H_j\right)^2 / \left(2a_j\right) \rightarrow min \qquad (4)$$

In practice, the objective function can be more complex with the introduction of additional elements.

III. NUMERICAL EXPERIMENTS AND RESULTS

In the numerical experiment, a grid $d = [1600, 800]$ was used, where $N = 34$ pollution emission sources are located (Fig. 1). The colors used represent the zones ($w = 8$) with different ecological standards, $\boldsymbol{Norm}$. The parameters were randomly obtained, as shown in Table 1. Some zones may contain pollution from up to three neighboring sources (e.g., N18 and possibly 2 and 12).

For simplification, the vector $\boldsymbol{c} = [1]$**,** was used, although in reality, it could have economic significance in the optimization results.

Table 1. Initial parameters of sources and one-dimensional optimization results. ($\boldsymbol{do} = \mathbf{0}$).

Source: authors.

n	Start					*SIP*. Iterations						
	$\boldsymbol{u_0}$	$\boldsymbol{H}$	$\boldsymbol{tN_1}$	$\boldsymbol{tN_2}$	$\boldsymbol{a}$	$\boldsymbol{u_1}$	$\boldsymbol{u_2}$	$\boldsymbol{u_3}$	$\boldsymbol{u_4}$	$\boldsymbol{u_5}$	$\boldsymbol{u_6}$	$\boldsymbol{u_7}$
1		11215	1566	69	0,11	0,59	1	1	0,91	0,84	0,87	0,87
2		7987	947	338	0,19	0,40	0,06	0,47	0,59	0,70	0,67	0,67
3		12613	504	219	0,13	0,39	0,59	0,61	0,71	0,82	0,82	0,82
4		1320	1221	704	0,12	0		0,50	0			
5		7986	309	630	0,09	0,59	0,28	0,77	0,55	0,60	0,67	0,67
6		4173	353	713	0,20	0,37	0,96	0,20	0,44	0,35	0,22	0,22
7		11081	1291	346	0,12	0,55	0,29	0,77	0,67	0,71	0,79	0,79
8		2974	1174	405	0,18	0,02	1			0,24	0,91	0,91
9		6177	1301	292	0,15	0,63	0,73	0,59	0,48	0,22	0,59	0,59
10		5552	128	479	0,11	0,55	1	0,29	0,61	0,54	0,44	0,44
11		11066	1463	163	0,19	0,38	0,62	0,62	0,72	0,78	0,76	0,76
12		9160	701	605	0,17	0,49	0,02	0,88	0,59	0,97	0,78	0,78
13		3186	73	761	0,13	0,07	0			0,45	0,45	0,45
14		4593	1229	453	0,18	0,02	1	0,42	0,83	1	0,60	0,60
15	0	2247	304	405	0,19	0,07	0	0,50	1	0	0,28	0,28
16		9091	837	721	0,12	0,40	1	0,73	0,79	0,46	0,79	0,79
17		7812	1374	774	0,11	0,75	0,07	0	0,74	0,74	0,74	0,74
18		2704	1094	330	0,15	0,02	1					
19		2423	1502	390	0,19	0,07	0		0,85	0,65	0,01	0,01
20		6604	381	324	0,16	0,56	0,30	0,73	0,74	0,53	0,74	0,74
21		12104	1136	453	0,11	0,30	1	0,78	0,72	0,80	0,72	0,72
22		7573	1057	775	0,17	0,01	0	0,12	0,75	0,75	0,75	0,75
23		962	1546	434	0,20	0			1		0,02	0,02
24		1229	1548	101	0,14	0,02	1					
25		10792	93	251	0,14	0,35	0,84	0,84	0,84	0,84	0,84	0,84
26		6289	936	431	0,10	0,63	1	0,90	0,87	0,57	0,76	0,76
27		5414	1448	439	0,07	1		0	0,45	0,54	1	
28		10596	700	440	0,09	0,96	0,19	0,75	1		0,83	0,83
29		5181	1147	22	0,07	0		0,91	0,62	1	0,33	0,33
30		7320	1288	122	0,13	0,69	0	0,29	0,75	0,64	0,69	0,69
31		9603	774	213	0,06	1		0,61	0,93	0,84	0,86	0,86
32		11638	600	535	0,19	0,29	1	0,53	0,72	0,71	0,86	0,86
33		4728	282	231	0,15	0,72	0	0,95	0,67	0,67	0,37	0,37
34		8820	327	164	0,06	0,97	1	0,40	1		0,84	0,84

The multi-extremal function describes the interaction of the set of sources at the beginning (Fig. 2). Changes in pollutant levels are observed, and they are not uniform in different directions. The maxima are located in the north and shift to the southwestern part of the map, most likely conflicting with the standards of zones 2, 3, 5, and 6. Naturally, in conflicts, it is necessary to consider the entire set of sources, regardless of their belonging to the zones of the region. In the right section of Table 1, the results of one-dimensional optimization using the *SIP* algorithm with seven iterations are shown. Based on the found maxima, the solution u indicates where pollution (source powers) should be reduced due to conflicts with existing standards.

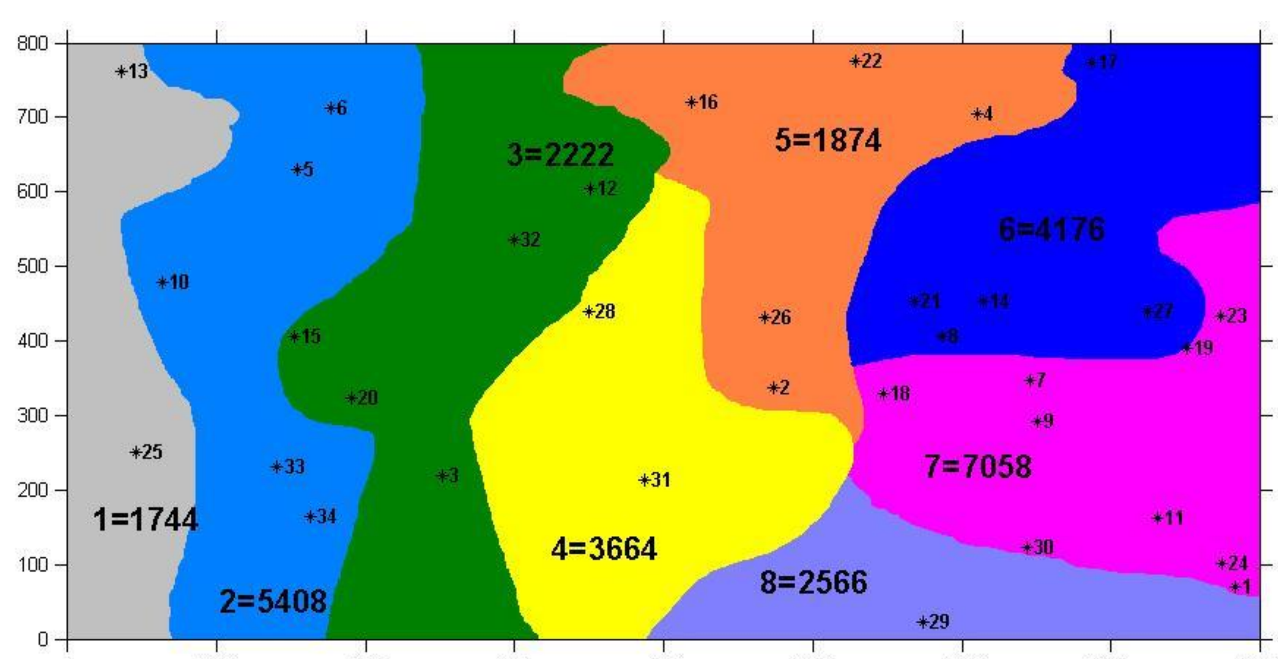

Fig. 1. Location of sources and area norms.

Source: authors.

For example, after iteration 3, sources 13, 17, 19, 23, and 27 retained their initial power ($\boldsymbol{u} = 0$). On the contrary, sources 1 and 8 should completely stop emitting ($\boldsymbol{u} = 1$). For the remaining sources, different reduction coefficients were obtained ($0 < \boldsymbol{u} < 1$). The solutions $\boldsymbol{u}$ may change from iteration to iteration and eventually stabilize, all depending on the conflict and algorithm. For instance, source 27 was the only one recommended for closure after seven iterations, even though it had an average reduction before that and returned to its full power in iteration 3.

In the one-dimensional *SIP* algorithm, the parameter $do = 0$. The second algorithm, *SIP2*, was executed in the range $do = [0{,}15;\ 0{,}75]$. The numerical results with $do = 0{,}45$ are shown in Table 2.

Table 2. Multidirectional optimization results. ($\boldsymbol{do} = \mathbf{0{,}45}$).

Source: authors.

<table>
<tr><th rowspan="2">n</th><th colspan="7">Iterations</th></tr>
<tr><th>u_1</th><th>u_2</th><th>u_3</th><th>u_4</th><th>u_5</th><th>u_6</th><th>u_7</th></tr>
<tr><td>1</td><td>0,41</td><td>0,57</td><td>0,05</td><td>0,62</td><td>0,61</td><td>0,56</td><td>0,56</td></tr>
<tr><td>2</td><td>0,18</td><td>0,41</td><td>1</td><td>0,29</td><td>0,51</td><td>0,63</td><td>0,64</td></tr>
<tr><td>3</td><td>0,31</td><td>0,58</td><td>0,45</td><td>0,58</td><td>0,56</td><td>0,59</td><td>0,59</td></tr>
<tr><td>4</td><td>0,01</td><td colspan="2">0</td><td>0,50</td><td colspan="3">0</td></tr>
<tr><td>5</td><td>0,07</td><td>0,62</td><td>0,27</td><td>0,64</td><td>0,65</td><td>0,57</td><td>0,56</td></tr>
<tr><td>6</td><td>0,09</td><td>0,51</td><td>1</td><td>0,30</td><td>0,28</td><td>0,43</td><td>0,46</td></tr>
<tr><td>7</td><td>0,70</td><td>0,46</td><td>1</td><td>0,57</td><td>0,77</td><td>0,71</td><td>0,67</td></tr>
<tr><td>8</td><td>0,76</td><td>0,83</td><td colspan="5">1</td></tr>
<tr><td>9</td><td>0,99</td><td>0,93</td><td>0,49</td><td>0,71</td><td>0,72</td><td>0,72</td><td>0,72</td></tr>
<tr><td>10</td><td>0,08</td><td>0,42</td><td>0,92</td><td>0,40</td><td>0,48</td><td>0,65</td><td>0,65</td></tr>
<tr><td>11</td><td>0,26</td><td>0,28</td><td>1</td><td>0,43</td><td>0,44</td><td>0,49</td><td>0,49</td></tr>
<tr><td>12</td><td>0,31</td><td>1</td><td>0,61</td><td>0,91</td><td>1</td><td>0,80</td><td>0,80</td></tr>
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<tr><td>15</td><td>0,01</td><td>0,25</td><td>0</td><td>0,50</td><td>0</td><td>0,06</td><td>0</td></tr>
<tr><td>16</td><td>0,64</td><td>0,05</td><td>0,40</td><td>0,79</td><td>0,79</td><td>0,79</td><td>0,79</td></tr>
<tr><td>17</td><td>0,07</td><td>0,36</td><td>0</td><td>0,09</td><td>0,01</td><td>0</td><td>0</td></tr>
<tr><td>18</td><td>0</td><td>0,67</td><td>0,16</td><td>0,43</td><td>0,31</td><td>0,75</td><td>0,68</td></tr>
<tr><td>19</td><td>0,02</td><td>0,50</td><td>0</td><td>0,35</td><td>0</td><td>0,01</td><td>0,01</td></tr>
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<tr><td>22</td><td>0,10</td><td>0</td><td>1</td><td>0,52</td><td>0,52</td><td>0,52</td><td>0,52</td></tr>
<tr><td>23</td><td colspan="4">0</td><td>0,04</td><td>0,02</td><td>0,02</td></tr>
<tr><td>24</td><td colspan="2">0</td><td colspan="5">1</td></tr>
<tr><td>25</td><td>0,45</td><td>0,58</td><td>0,58</td><td>0,58</td><td>0,58</td><td>0,58</td><td>0,58</td></tr>
<tr><td>26</td><td>0,53</td><td>0,77</td><td>1</td><td>0,82</td><td>1</td><td>0,65</td><td>0,65</td></tr>
<tr><td>27</td><td colspan="7">1</td></tr>
<tr><td>28</td><td>0,94</td><td>0,59</td><td>0,30</td><td>0,52</td><td>0,74</td><td>0,85</td><td>0,85</td></tr>
<tr><td>29</td><td>0</td><td>0</td><td>0</td><td>0,04</td><td>0</td><td>0,12</td><td>0,53</td></tr>
<tr><td>30</td><td>0,63</td><td>0,74</td><td>0</td><td>0,47</td><td>0,51</td><td>0,50</td><td>0,57</td></tr>
<tr><td>31</td><td>1</td><td>0,76</td><td>1</td><td>0,82</td><td>0,55</td><td>0,89</td><td>0,89</td></tr>
<tr><td>32</td><td>0,26</td><td>0,69</td><td>1</td><td>0,76</td><td>0,69</td><td>0,85</td><td>0,85</td></tr>
<tr><td>33</td><td>0,93</td><td colspan="6">1</td></tr>
<tr><td>34</td><td>1</td><td>0,80</td><td>0,87</td><td>0,87</td><td>0,87</td><td>0,87</td><td>0,87</td></tr>
</table>

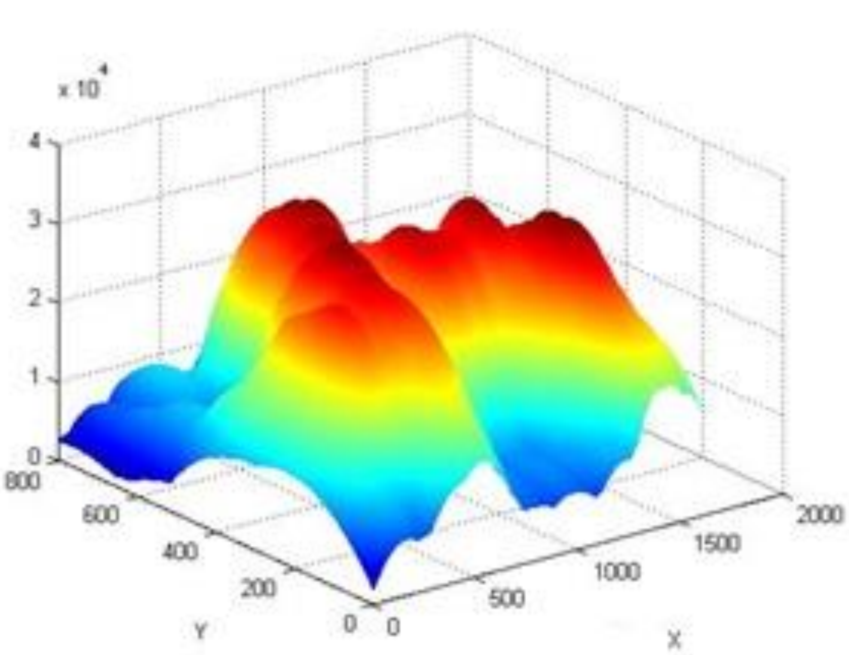

Figura 2. Total pollution function of the area.

Source: authors.

The results of the two algorithms show different roles of the sources in achieving total pollution and, as a consequence, the need for changes in their emissions. The visual analysis of the solution $\boldsymbol{u}$ for the entire set of sources is complicated. The new solution $\boldsymbol{u}$ alters the vector $\boldsymbol{H}$ and its structure towards the vector $\boldsymbol{Norm}$. For the function (3), the following must hold:

$$pre_{j,tN.u} = \left(1 - u_j\right)H_j - Norm_{j,tN} \qquad (5)$$

In Fig. 3, the total change (5) from one-dimensional optimization is shown in black. Initially, there are immense and prevailing excesses.

The value of $\text{pre} = 0$ is achieved after the first iteration, and in the subsequent iterations: $\text{pre} \ll 0$. For the given conflict, the best combination of source powers is found in the $do = 0{,}35 - 0{,}55$ interval.

Fig. 3 justifies the chosen value of $do = 0{,}45$ based on the pollution function excesses over the norm vector. Fig. 4 shows the characteristics of solutions with one-dimensional and multidirectional optimization.

It can be observed that to comply with the $\boldsymbol{Norm}$ standards, the *SIP* algorithm needs to decrease industrial powers almost 6 times (black color in Fig. 3). However, for *SIP2* (color in Fig. 3), the pollution reduction is slower. For example, with $do = 0{,}45$ starting from iteration 4, there are reductions of 2.5–3 times.

Figs. 5 and Fig. 6 allow for a comparison of the two graphs corresponding to the *SIP* and *SIP2* algorithms, respectively. Compared to the initial state (Fig. 2), both functions show a decrease in considered pollution in the studied area. However, the optimization with *SIP* still shows maxima up to 5000–6000 units (Fig. 5), while the optimization with *SIP2* has visibly larger maxima, up to 9000–10000 units (Fig. 6), which may not be immediately perceived as an improvement.

Despite everything, it is preferable to use the function shown in Fig. 6, where the most significant gradients are observed, with zero levels in the surroundings, characterized by negative values of the criterion pre. Moreover, the pollution reduction of some sources is accompanied by the increase of others. Most importantly, the overall pollution value in Fig. 6 is more consistent with the pollution norms vector of the area, $\boldsymbol{Norm}$.

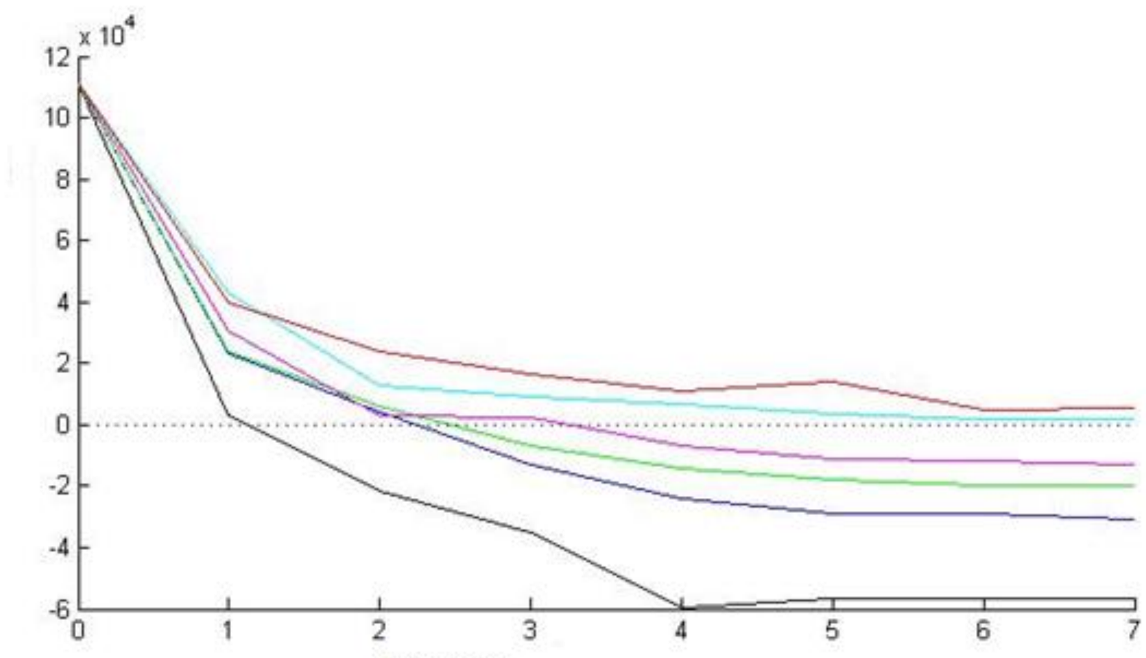

Fig. 3. Changes in total emissions from sources.

Source: authors.

Table 3. The overall pollution of the area $\boldsymbol{Ost_{u,do}}$ according to the iterations.

Source: authors.

do	Iterations						
	1	2	3	4	5	6	7
0	0,37	0,33	0,17	0,09	0,10	0.08	0,08
−0,35	0,49	0,34	0,32	0,27	0,26	0,26	0,24
−0,55	0,57	0,39	0,34	0,32	0,36	0,31	0,31
−0,75	0,56	0,47	0,41	0,43	0,39	0,36	0,37

In Table 3, the given numerical experiment of the 7 optimization iterations is not complete, but its essential part is visible. Essentially, with the *SIP2* algorithm, there is an exchange of pollutant powers between sources to approach the norm vector better.

If applying the *SIP2* algorithm becomes challenging for some reasons, it is possible to carry out the scheme with fewer iterations using the parameter do (e.g., after 3, 5, or fewer iterations). For each iteration, a new solution $\boldsymbol{u}$, was used, while the vector $\boldsymbol{H}$ was preserved.

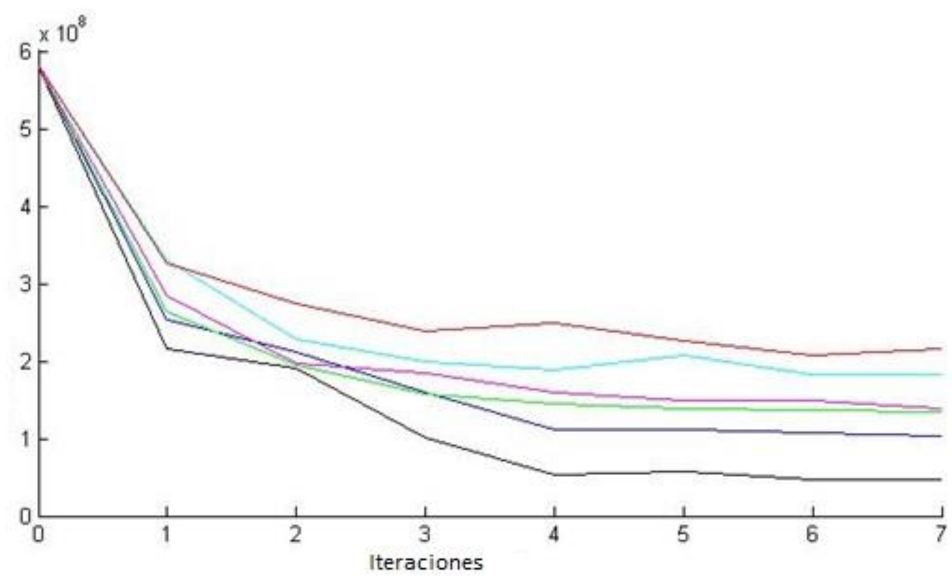

Figure 4. Change in total pollution.

Source: authors.

The new *SIP* version preserves the type and characteristics of the total emission function. The norms not only indicate where there is excess pollution but also dictate the allowances for its increase in some sources. This way, the pollution reduction optimization is performed in complete balance with the norms.

Unlike problems of searching for global maxima, the semi-infinite optimization algorithms aim to find all critical points (local maxima).

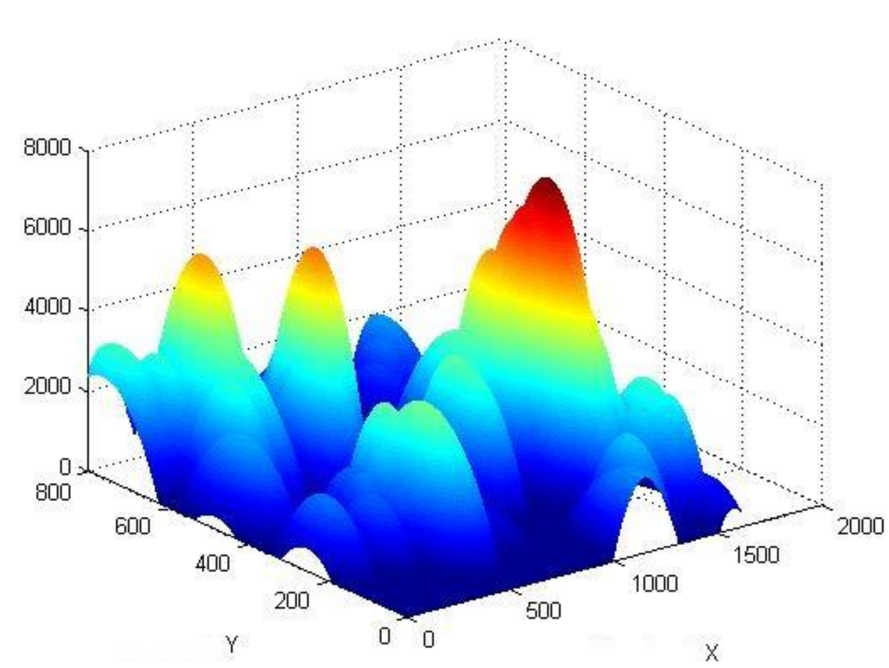

Fig. 5. Total pollution function after one-dimensional optimization (do=0, u3).

Source: authors.

IV. Conclusions

With the proposed algorithm, the conflicts presented are resolved using multi-extremal functions where optimization is carried out in two directions. This approach allows for both reducing pollution and identifying areas where it could coexist and have a higher level if needed. By considering both reduction and potential increase in pollution, the algorithm achieves a more balanced and comprehensive optimization solution, ensuring compliance with environmental norms while also maximizing productivity and industrial potential.

The application of the *SIP2* version allows for solutions such as locating a powerful complex on the plane that complies with environmental norms and provides more possibilities for conflict resolution.

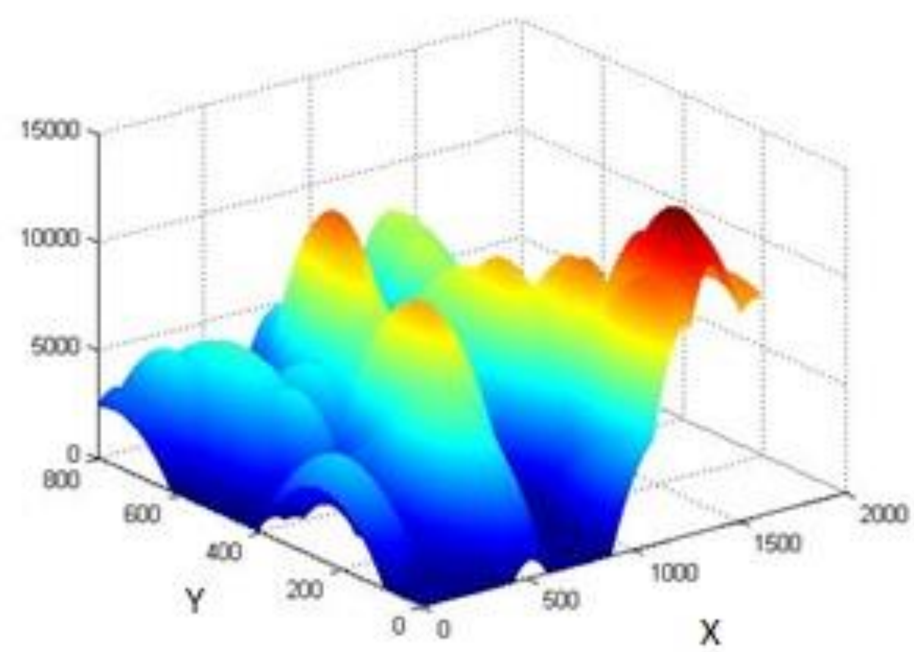

Fig. 6. Total pollution function after multidimensional optimization (do=0.45, u4).

Source: authors.

V. CRediT authorship contribution statement

Alina Fedossova: Software, Writing-Revision and editing.

Valery Fedosov: Conceptualization, Research, Writing-Original Draft.

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